Spring 2020

MATH 5A - TEST 2
(2.2-2.6, 2.8, 2.9)

100 points **NAME:**_______________

FILL IN THE BLANKS WITH MOST APPROPRIATE ANSWER: (2 points)

(1) If V(t) represents the volume water in the bath tub (in cubic inches) at time t where t is the number of minutes after 6:00 p.m., explain very specifically words, with units, what $\left.\frac{dV}{dt}\right|_{t=3}$ represents

The instantaneous rate of change of volume of water in tub relative to time at 6:03pm. Units in^3/min.

(2) If $y = \tan x$, the differential, $dy =$ $\sec^2 dx$.

(3) $\lim_{x\to 0} \frac{\sin x}{x}$ 1

(4) If $f(x) = 3x^5$ then $f''(x) =$ $60x^3$

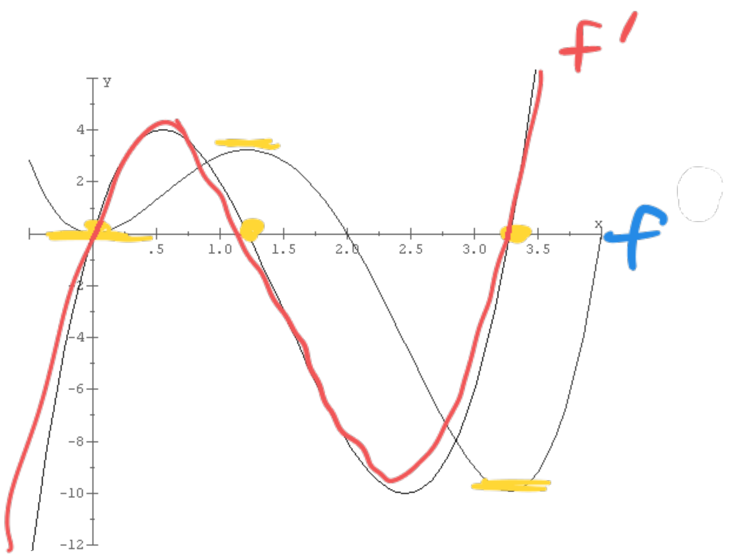
$f' = 15x^4$

(5) True or False: If f is differentiable at x=a then f is continuous at x=a. True

(6) Given that $f(x) = g(x^2) + [g(x)]^2$, find $f'(x)$. (3 points)

$f'(x) = g'(x^2)\,2x + 2g(x)g'(x)$

(7) The graphs below are of a function and its derivative. Clearly label which is $f(x)$ and which is $f'(x)$. (4 points)



In problems 8- 12 , find $\frac{dy}{dx}$. Work carefully, very <u>limited</u> partial credit will be given. Simplify your answers. Do not leave any negative exponents or complex fractions. Combine fractions(8 pts each)

(8) $y = \sqrt{x}\left(x^2+3\sqrt{x}\right) = x^{5/2} + 3x$

$\boxed{\frac{dy}{dx} = \frac{5}{2}x^{3/2}+3}$

(9) $y = \sin\left(\frac{x^2}{2x+1}\right)$

$y' = \cos\left(\frac{x^2}{2x+1}\right)\frac{d}{dx}\left(\frac{x^2}{2x+1}\right)$

$y' = \cos\left(\frac{x^2}{2x+1}\right)\frac{(2x+1)2x - x^2(2)}{(2x+1)^2}$

$\boxed{y' = \cos\left(\frac{x^2}{2x+1}\right)\frac{2x^2+2x}{(2x+1)^2}}$

(10) $y = \frac{x^2}{\sqrt{9-x^2}} = x^2(9-x^2)^{-1/2}$ (use or quotient rule)

$y' = 2x(9-x^2)^{-1/2} + x^2 \cdot -\frac{1}{2}(9-x^2)^{-3/2}(-2x)$

$y' = 2x(9-x^2)^{-1/2} + x^3(9-x^2)^{-3/2}$

$y' = x(9-x^2)^{-3/2}\left(2(9-x^2)+x^2\right)$

$\boxed{y' = \frac{x(18-x^2)}{(9-x^2)^{3/2}}}$

(11) $y = \cos^3\left(\sqrt{x}\right) = (\cos\sqrt{x})^3$

$y' = 3(\cos\sqrt{x})^2\frac{d}{dx}\cos\sqrt{x}$

$y' = 3\cos^2\sqrt{x} \cdot -\sin\sqrt{x}\,\frac{d}{dx}\sqrt{x}$

$\boxed{y' = \frac{-3\cos^2\sqrt{x}\,\sin\sqrt{x}}{2\sqrt{x}}}$

(12) $\sin(xy)=y^2$

$\frac{d}{dx}\sin(xy) = \frac{d}{dx}y^2$ implicit diff.

$\cos(xy)\frac{d}{dx}(xy) = 2y\frac{dy}{dx}$

$\cos(xy)\left(y + x\frac{dy}{dx}\right) = 2y\frac{dy}{dx}$

$y\cos(xy) + x\cos(xy)\frac{dy}{dx} = 2y\frac{dy}{dx}$

$y\cos(xy) = 2y\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx}$

$y\cos(xy) = \frac{dy}{dx}(2y - x\cos xy)$

$\boxed{\frac{dy}{dx} = \frac{y\cos(xy)}{2y - x\cos xy}}$

(13) Use differentials or linear approximation to approximate $\sqrt[3]{26.96}$ (9 points)

Let $f(x) = x^{1/3}$, approximate $f(26.96)$

let $a = 27$

$f(27) = 27^{1/3} = 3$

$f'(x) = \frac{1}{3x^{2/3}}$

$f'(27) = \frac{1}{27}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = f(27) + f'(27)(x-27)$

$L(x) = 3 + \frac{1}{27}(x-27)$

$f(26.96) \approx L(26.96) = 3 + \frac{1}{27}(26.96-27)$

$= 3 + \frac{1}{27}(-.04) = \boxed{2.9985}$

Calculator: 2.998518

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(14) Find the x values of the points on the curve $y = \frac{\cos x}{2+\sin x}$ at which the tangent is horizontal. (9 pts)

Find where $f'(x) = 0$

$$y' = \frac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2}$$

$$y' = \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$y' = \frac{-(2\sin x + \underbrace{\sin^2 x + \cos^2 x}_{})}{(2+\sin x)^2}$$

$$y' = \frac{-2\sin x - 1}{(2+\sin x)^2}$$

Horiztontal tangent when $y' = 0$

$\Rightarrow -2\sin x - 1 = 0 \quad \sin x = -\frac{1}{2}$

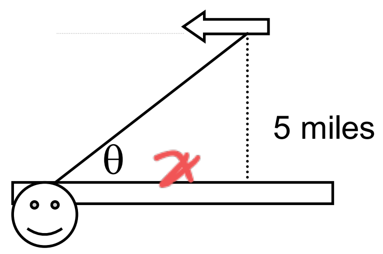
$x = \frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$

(15) An airplane flies at an altitude of 5 miles directly toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation, θ, is changing when the angle is 30°.(show units)

Know $\frac{dx}{dt} = -600$

Want $\left.\frac{d\theta}{dt}\right|_{\theta=30^\circ}$

(8 points)



$$\tan\theta = \frac{5}{x}$$

$$\frac{d}{dt}\cot\theta = \frac{d}{dt}\frac{x}{5}$$

$$-\csc^2\theta\,\frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = -\frac{1}{5}\frac{dx}{dt}\sin^2\theta$$

$$\left.\frac{d\theta}{dt}\right|_{\theta=30^\circ} = -\frac{1}{5}(-600)(\sin 30)^2$$

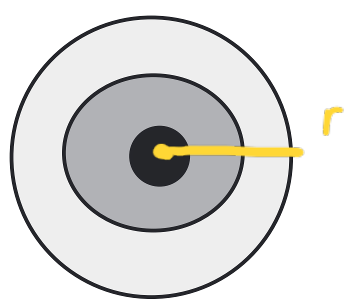
$$= \boxed{30 \text{ degrees/hr}}$$

(16) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 cm/sec. How fast is the area enclosed by the ripple increasing when t=5 seconds? (show units)

Know $\frac{dr}{dt} = 3\frac{cm}{sec}$

Want $\left.\frac{dA}{dt}\right|_{t=5}$

(8 points)



notice units

$r = 3t$

At $t=5$ sec

$r = 15$

$$A = \pi r^2$$

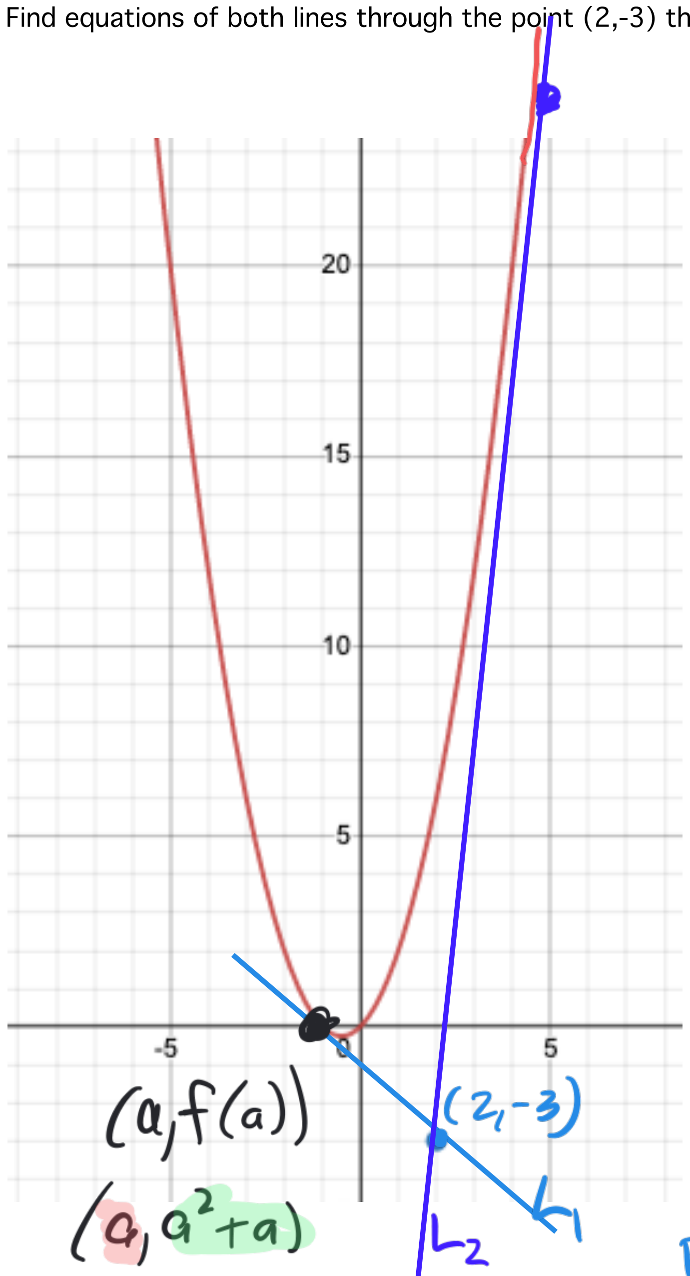
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{t=5} = 2\pi(15)\,3 = \boxed{90\pi \text{ cm}^2/\text{sec}}$$

(17) Find equations of both lines through the point (2,-3) that are tangent to $f(x) = x^2 + x$.

$f'(x) = 2x+1$

(9 points)



Caution:
In this problem, we are not given the point of tangency

Let $P(a, a^2+a)$ be point of tangency

Find ~~equation~~

$m = f'(a)$
$= 2a+1$

point $(2,-3)$

$y+3 = (2a+1)(x-2)$

P is on line also, so substitute

$a^2+a+3 = (2a+1)(a-2)$

Now solve for a

$a^2+a+3 = 2a^2-3a-2$

$0 = a^2-4a-5$

$0 = (a-5)(a+1)$

$a = 5, -1$

L1 $\quad a=-1, \; f'(-1) = -1 = m$

$y+3 = -1(x-2)$

$y = -x-1$

L2 $\quad a = 5 \quad f'(5) = 11 = m$

$y+3 = 11(x-2)$

$y = 11x-25$